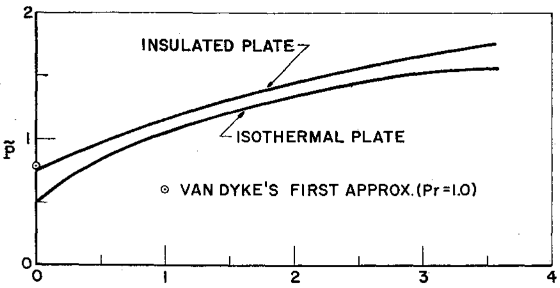

Fig. 1 Induced pressures for the initial stage case, with $U(t) = At^m$;

$$\tilde{P} \equiv \left(\frac{P_b}{P_\infty} - 1\right) \Big/ \left[\frac{\gamma(\gamma - 1)}{2} M^2 R^{-1/2}{}_*\right]$$

will be studied, using the results of the boundary-layer solutions recently reported by Yang and Huang.[6] Note here that in applying Eq. (6), only δ_ρ is relevant. This implies that the induced pressure in this case results solely from density defects inside a boundary layer. The induced pressures have the following form:

$$(P_b - P_\infty)/P_\infty = [\gamma(\gamma - 1)/2](\tfrac{1}{2} + m)K(m)M^2/(R^*)^{1/2} \sim t^{(2m-1/2)} \quad (10)$$

where $R^* \equiv a_\infty{}^2 t/\nu$ and $K(m)$ is a known constant, with different values for insulated and isothermal plates, for each given value of m. Results of Eq. (10) are illustrated in Fig. 1.‡

We note here that the form of the induced pressure in this limit is the same as that given by Van Dyke[7] who treated a case equivalent to $m = 0$ in our results but with $Pr = 1$. Also it is interesting to note that $m = \frac{1}{4}$ is a distinguished case for which the induced pressure, to first approximation, is constant in time.

In conclusion, we remark that the unsteady, weak interaction pressures on a flat plate have been worked out for both the case of small flow unsteadiness and the case of initial stages of motion (or equivalently, the case of a doubly infinite flat plate). It has also been indicated that the idea based on which the method of calculations reported in this Note is developed may probably be employed to pursue the study of the conceivably more complex problem of strong pressure interactions in unsteady flows.

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‡ Errors in the numerical values of Table 1 in Ref. 6 have been noticed, and corrected values were obtained through private communications.

Experimental Verification of St. Venant's Principle in a Sandwich Beam

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St. VENANT'S principle is well known and has been demonstrated theoretically and also experimentally. From this principle it is understood that the difference between the stresses caused by two statically equivalent load systems is insignificant at and beyond a distance equal to the largest linear dimension of the area over which these load systems are applied. But it has also been shown in some cases such as thin walled structures[1] and space trusses[2] that a self-equilibrating load induces significant stresses even at large distances from the end. In other words, in such cases the effect of replacing one load system by another statically equivalent load system is felt significantly at large distances beyond the area of application of the load.

In this Note one such case, namely the problem of a sandwich beam subjected to two different statically equivalent load systems, has been investigated, and it has been shown that the difference between the stresses caused by these two statically equivalent load systems at a distance equal to the depth of the beam and beyond is much larger than what it would be in the case of a homogeneous beam.† Also it is shown that this is true only under a certain condition, viz., when the Young's modulus of the core is quite small compared to the Young's modulus of the face layers. The author, attempting to give exact solutions by elasticity theory to sandwich plates, found it necessary to use the St. Venant's principle in order to satisfy the boundary conditions at the thickness edge of the plate. In this connection, it was thought worthwhile to investigate how valid this principle is in the case of a sandwich beam. It can also be mentioned that this principle is assumed or used indirectly in many cases connected with analysis of sandwich structures. A better understanding of this problem will lead to a more judicious application of this principle in the field of sandwich structural analysis.

As a first step in this process, experiments were carried out on a simple sandwich cantilever beam using a photoelastic technique. The sandwich beams were 25 cm long and 5 cm deep. The depth of each of the layers was 2 cm and the depth of the core was 1 cm. The width of the beam was 1.5 cm. In order to prevent the lateral buckling of the low modulus core it was necessary to keep the width of the beam considerably large, although this introduced additional difficulties in accurate measurement of the fringe order at the

Table 1 Model materials

Case 1	Face layers	Araldit D (epoxy resin)
	core layer	Silicone Kautschuk Gießmasse (rubber)
Case 2	Face layers	Araldit D
	core layer	Araldit D + 45./.Thiokol (plasticizer)
Case 3	Homogeneous beam	

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† It is possible that one can arrive at this conclusion with certain amount of reasoning. But the object of this investigation is to give a quantitative estimate of this effect.

Table 2 Ratios of Young's Modulus

Case 1	E_{face}/E_{core}	3600
Case 2	E_{face}/E_{core}	9
Case 3	E_{face}/E_{core}	1

edges. The difficult part of the experimental study was to find materials of highly different elastic properties, which have good adhesion with each other and at least one of which is transparent and photoelastically sensitive. After a considerable number of trial experiments the sandwich models, shown in Table 1, were constructed and the test results analysed.

In the first two cases the core material was cast integral with Araldite strips so that good adhesion was obtained at the interface. Although Silicone Kautschuk is not transparent, it has a low Young's modulus, significantly good adhesion with Araldite and favorable creep characteristics. The ratios, viz., E_{face}/E_{core}, for the three cases are shown in Table 2.

Each model was loaded as a cantilever with a concentrated load at the end. The concentrated load was applied at the same cross section but at different points A and B (see Fig. 1) so that they formed two statically equivalent load systems. In each case the isochromatics were recorded and the exact magnitudes of the fringe orders were determined by interpolation. Each picture was taken 20 min after loading so that the creep effects were negligible. The fringe order distribution, which is proportional to the maximum shear-stress distribution, has been compared for the two loading systems at different distances from the end. Since the interest was mainly in finding out to what distance the disturbance is carried over, it was thought that any particular type of stress would be sufficient to indicate the effect; hence the complete analysis of the principle stresses was not attempted.

The results are given in Fig. 2. For the first case, the distribution of fringe order is plotted across the cross sections at various distances, b, $2b$, $3b$, from the end. In the second case, for the same depth of the beam, the fringe order could be determined with reasonable degree of accuracy at several points only at distances $2b$ and $3b$ from the end. From the results of Fig. 2, it can be seen that in the first case, where the Young's modulus of the core is very low, at a distance b from the end, the difference between the two stresses caused by the two statically equivalent load systems was as high as 40%; whereas in the case of a homogeneous beam, at a distance equal to the depth from the end, the difference was found to be nowhere more than 5%. It has been shown theoretically by Bleich[3] that in the case of a homogeneous semi-infinite strip at a depth equal to the width from the loaded end, the difference between the stress caused by a concentrated load and uniformly distributed load is not more than 3%.

Again from Fig. 2, it can be seen that at a distance $2b$ the maximum difference is about 18% in case 1. From Table 3 it can be seen that it is around 6% in case 2. At a distance $3b$ the difference is almost insignificant. It is possible that a significant difference between two stress systems would exist even at a distance $3b$ and beyond but for the fact that the fixed edge condition which is the same for the two load systems has an equalizing influence. In the case of a longer beam and with a core of much lower modulus of elasticity the disturbance stresses may be carried over for much larger distances. In terms of ratios, it can be seen from Fig. 2 that at a distance b the stress due to one load system may be nearly 1.7 times the stress due to another statically equivalent load system. It has been shown by Salerno[1] that in the case of a thin walled monocoque structure this ratio at a distance equal to the diameter of the shell may be as high as 2.2.

In this connection it should be mentioned that E_{face}/E_{core} ratios can take values up to about 6000 in the case of solid expanded core materials[4] which is twice the value that has been

Table 3 Fringe order distribution at a distance $2b$

Distance, mm	Load 1 Fringe order	Load 2 Fringe order
2	3.2	3.1
4	3.0	2.8
9	2.1	2.0
14.5	1.0	0.94
40	1.0	1.0
45	2.0	2.1
49.25	3.0	3.2

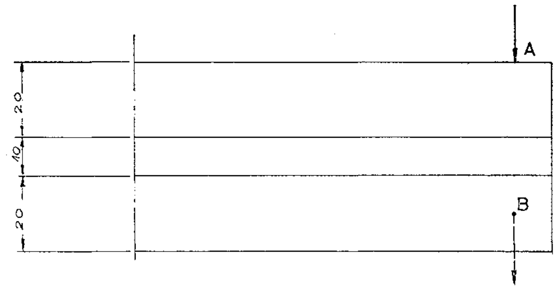

Fig. 1 Position of the loads.

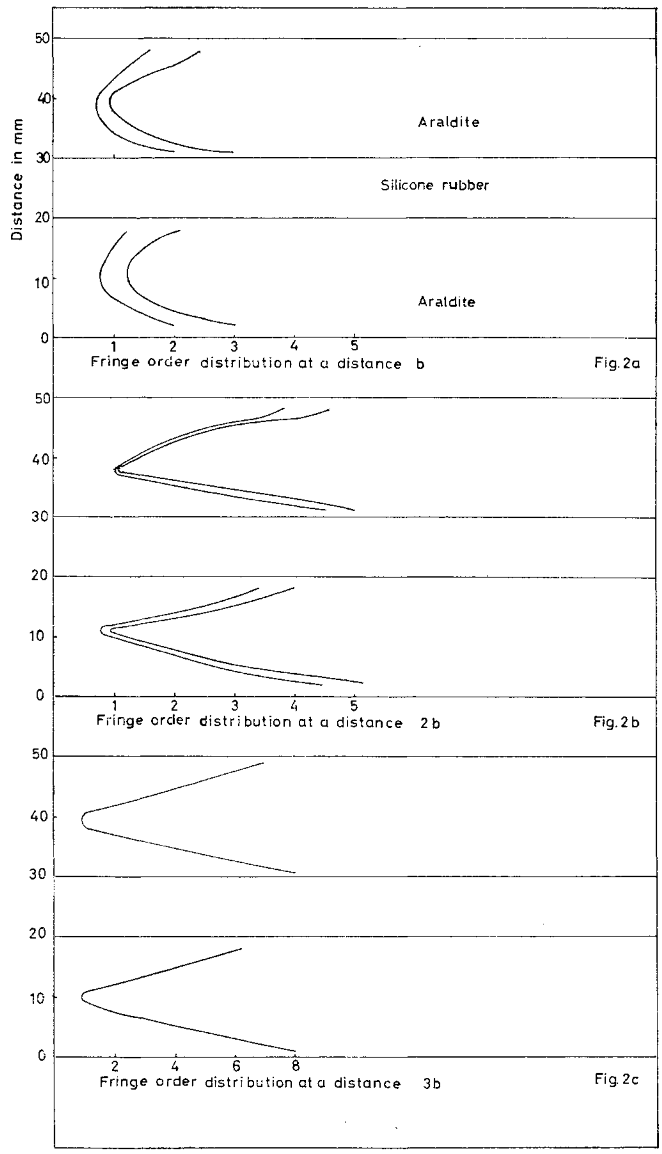

Fig. 2 Fringe order distribution (case 1).

considered here. At the same time, the core to face thickness ratio that has been considered here is much smaller than what would occur in a normal honeycomb sandwich construction.

At present the author is engaged in the theoretical investigation of a semi-infinite sandwich beam subjected to statically equivalent load systems, in order to find the effect of the various parameters in greater detail.

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Catalytic Effects of $KMnO_4$ on the Deflagration of Ammonium Perchlorate: A Preliminary Study

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Introduction

For ammonium perchlorate (AP) catalysts can significantly alter the rate of thermal decomposition, the burning rate, and the deflagration pressure limit, which is the inert gas pressure below which stable, unassisted deflagration is impossible. Various additives give widely differing effects which may be drastically changed by using different concentrations of the same catalyst. (See the review of these data by Pittman.[1]) Since the process of AP deflagration is not completely understood, many questions concerning the mechanism of catalyzed AP deflagration remain unresolved. This research was designed to investigate the importance of the degree of dispersion of the catalyst in the ammonium perchlorate and to investigate the correlation of catalytic effectiveness on AP thermal decomposition and in AP deflagration.

Most studies of catalysts for AP deflagration have used pellets pressed from mechanical mixtures of AP and catalyst, and very little work has been done to see if the catalyst particle size influences the catalytic effectiveness. Although there is growing evidence for a molten zone of AP at the burning surface,[2] it might still be expected that the surface area of the catalyst, rather than the total mass, controlled the reaction rate. Potassium permanganate was chosen for this catalytic study because it has the same crystal structure as ammonium perchlorate and can be isomorphically cocrystallized with ammonium perchlorate. In addition, the $KMnO_4$ was also mechanically mixed with the AP to observe the effects of catalyst dispersion on deflagration.

Considerable attention has been given to the thermal decomposition of AP single crystals doped with potassium permanganate[3] and of AP cocrystallized with potassium permanganate.[4–6] These studies[4,5] indicated that the rate of thermal decomposition increased as the fraction of $KMnO_4$ was increased from 0.5 mole % to 2 mole %. In addition, the activation energy for this process with 2 mole % $KMnO_4$ was found to be about 20 kcal/mole at all temperatures studied[4]; this value is extremely close to the value of 17–20 kcal/-mole determined by Bircumshaw and Newman[7,8] for AP above 240°C in the cubic modification. Below this temperature Bircumshaw and Newman[8] found the activation energy for the thermal decomposition of orthorhombic AP to be 28–29 kcal/mole. In all cases the $KMnO_4$ considerably accelerated the thermal decomposition of AP. At present it is not certain that the AP thermal decomposition is a step in the deflagration process; consequently a second objective of the present preliminary work was to compare the catalytic effects of $KMnO_4$ on AP deflagration with its reported effects on AP thermal decomposition.

Experimental Procedure

Large single crystals were grown with 0.4 mole % $KMnO_4$ isomorphically substituted in the AP lattice by the same method that has been used for growing pure AP single crystals.[9,10] The temperature of a saturated aqueous solution of AP and $KMnO_4$ was lowered very slowly, and small AP crystals were used as the initial seeds. From the resulting crystals perfect sections measuring about 0.4 cm on each side and 1.2 cm in length were cleaved for the deflagration studies. Imperfect crystals were powdered to make pellets with 0.4 mole % $KMnO_4$, and, for comparison, pellets were pressed with a mechanical mixture of 0.4 mole % $KMnO_4$. Pellets of 2 mole % $KMnO_4$ were also prepared from cocrystallized AP-$KMnO_4$ powder. In addition, a layered pellet was made by pressing pure AP powder on top of an AP-0.4 mole % $KMnO_4$ pellet; the pure section was ignited and this, in turn, ignited the AP-$KMnO_4$ section of the pellet.

The combustion chamber was pressurized with nitrogen and the deflagration was photographed with a 16-mm Bolex camera at 64 frames/sec. The samples were ignited by a jet of hot nitrogen impinging normal to the pellet or crystal surface. While the pellet or crystal surface was being heated by the igniter, thermal decomposition caused the surface to recede slowly, but considerably faster than during the ignition of pure AP. Unlike the more usual hot Nichrome wire ignition technique,[2,10,11] the hot nitrogen ignition gas was uniquely adapted to following the gradually receding surface during the ignition transient. A detailed description of the apparatus and procedure has been presented elsewhere.[11]

Discussion of Results

The single crystals of AP with 0.4 mole % $KMnO_4$ isomorphously substituted into the crystal lattice, the pellets pressed from powder with 0.4 and 2 mole % $KMnO_4$ in AP, and the pellets pressed from a mechanical mixture of 0.4 mole % $KMnO_4$ in AP were ignited at either 1000 or 2000 psig and all failed to sustain deflagration. High-speed motion photography indicated that with the hot gas impinging on the surface rapid decomposition would take place and occasional puffs of smoke would be emitted, but all reactions extinguished rapidly with the removal of the ignition gas jet.

In light of the catalytic effect on the $KMnO_4$ on the thermal decomposition of AP it seemed that while the hot ignition gas was striking the crystal or pellet surface the thermal decomposition provided a heat sink that prevented the surface temperature from reaching the ignition temperature. If deflagration is possible, a considerably stronger ignition stimulus may be necessary to overcome this heat sink effect. This consideration led to the experiments with the layered pellets in which the deflagrating AP section was used to ignite the catalyzed section of the pellet, and, again, the catalyzed pellet did not deflagrate. Although this preliminary work did not explore other ignition techniques, it should be noted that

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